NAPREDNE METODE TEHNOLOGIJE PLASTIČNOG DEFORMISANJA

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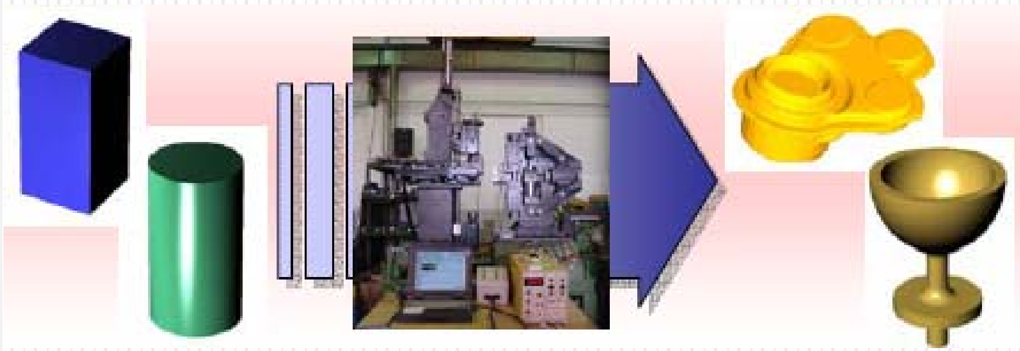
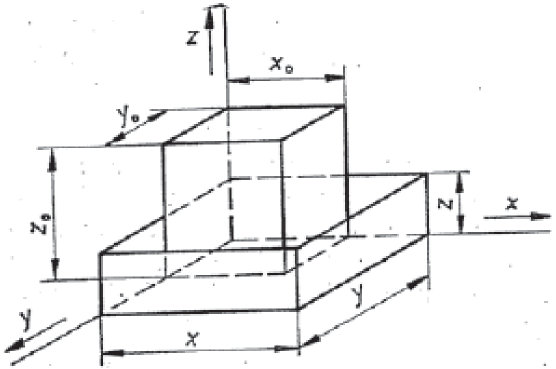
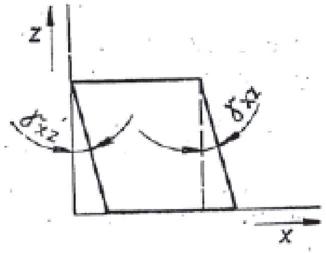


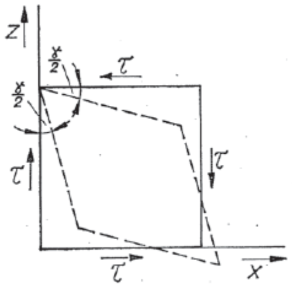
Pojam deformacije i načini njenog izražavanja

Deformisanje – proces plastičnog oblikovanja (promena dimenzija i oblika)
Deformacije – pokazatelj stepena izvršenog plastičnog deformisanja
Homogeno i nehomogeno deformisanje.
Male deformacije – deformacije koje odgovaraju manjim etapama deformisanja
Ukupne deformacije

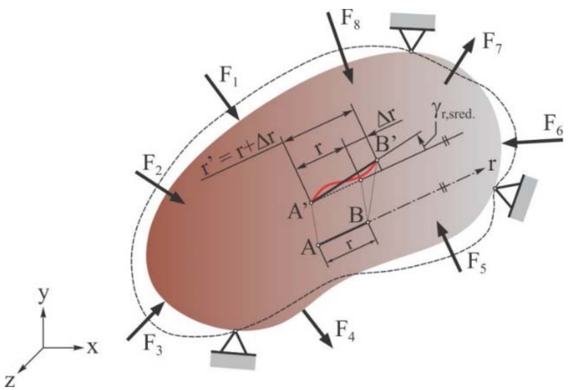
Linijske (ε) i ugaone (γ) deformacije

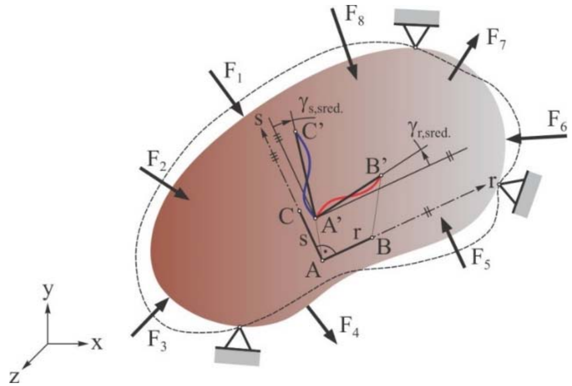






$$\varepsilon_x = \frac{\Delta x}{x_o} \qquad \varepsilon_y = \frac{\Delta y}{y_o} \qquad \varepsilon_z = \frac{\Delta z}{z_o}$$



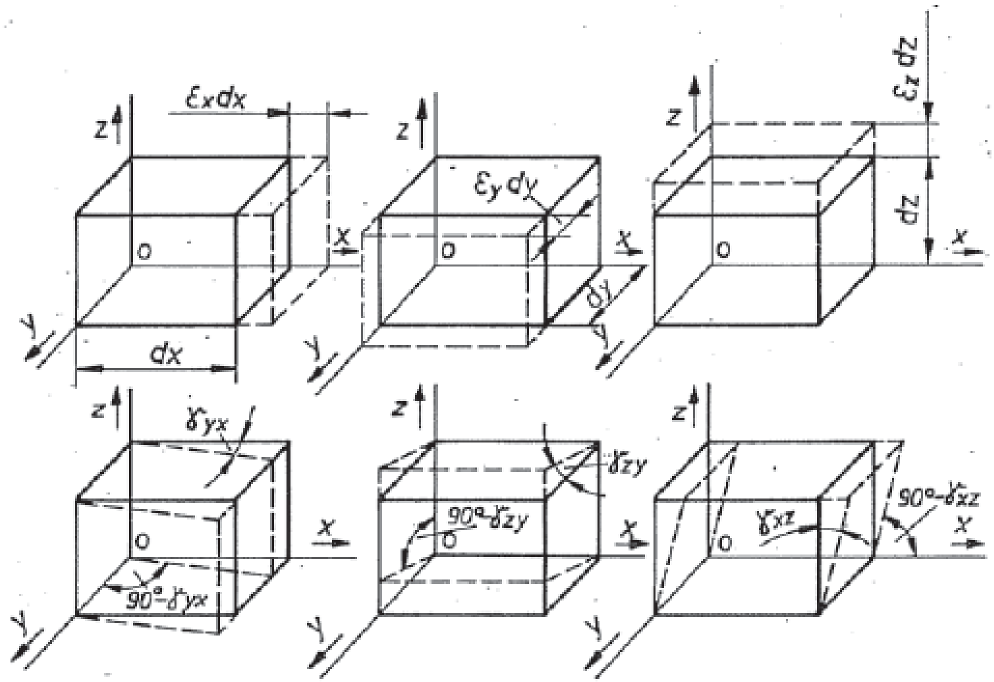


$$\boxed{\gamma_{r,sred.} + \gamma_{s,sred.} = \gamma_{rs,sred.}}$$

$$\boxed{\lim_{\substack{r\to 0\\ s\to 0}} \left(\gamma_{r,sred.} + \gamma_{s,sred.}\right) = \gamma_{rs} = \gamma_{sr}}$$

Pojam deformacije i načini njenog izražavanja

- Deformaciono stanje: poznate 3 linijske (ε_x, ε_y, ε_z) i 3 ugaone (γ_{xy}, γ_{yz}, γ_{zx}) deformacije
- Deformacije su u opštem slučaju funkcije koordinata!!!
- Pojednostavljenja – osrednjavanje deformacija (inženjerska praksa)
- Glavne deformacije $|\varepsilon_1| \geq |\varepsilon_2| \geq |\varepsilon_3|$ (γ=0).



Određivanje deformacije za čitavo telo može se vršiti samo ako su osnovni geometrijski oblici početnog i krajnjeg stanja isti (paralelopiped, cilindar, ...). U protivnom, moraju se ove deformacije svoditi na elemente tela, kao što se čini, na primer, pri analizi dubokog izvlačenja šupljeg cilindričnog predmeta.

Načini izražavanja deformacija u TPD

•*apsolutna deformacija*

$$\Delta l = l_0 - l$$

•

•*jedinična (relativna) deformacija*

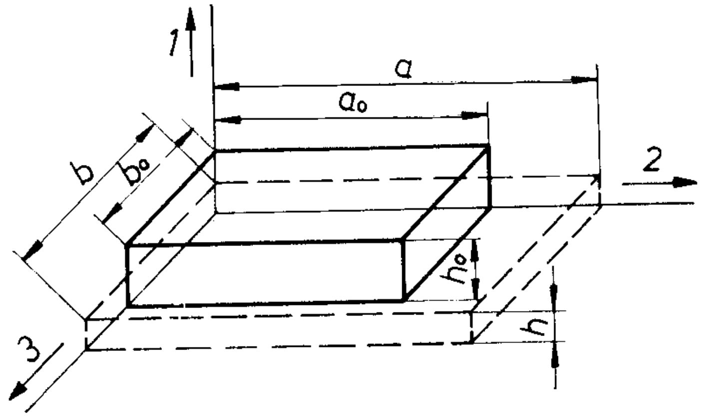
$$\varepsilon = \frac{\Delta l}{l_0} = \frac{l - l_0}{l_0} = \frac{l}{l_0} - 1$$

•*deformacija površine (poprečnog preseka)*

$$\psi = \frac{\Delta A}{A_0} = 1 - \frac{\Delta A}{A_0} = 1 - \frac{\Delta l}{l_0} = 1 - \frac{\varepsilon}{1+\varepsilon}$$

•*logaritamska (stvarna) deformacija*

$$d\varphi = \frac{dl}{l} \Rightarrow \varphi = \int_{l_0}^{l} \frac{dl}{l} = \ln\frac{l}{l_0}$$



$$\varepsilon_a = \frac{\Delta a}{a_0} = \frac{a - a_0}{a_0} > 0 \qquad \varphi_a = \int_{a_0}^{a} d\varphi_a = \int_{a_0}^{a} \frac{da}{a} = \ln\frac{a}{a_0} > 0$$

$$\varepsilon_b = \frac{\Delta b}{b_0} = \frac{b - b_0}{b_0} > 0 \qquad \varphi_b = \ln\frac{b}{b_0} > 0$$

$$\varepsilon_h = \frac{\Delta h}{h_0} = \frac{h - h_0}{h_0} < 0 \qquad \varphi_h = \ln\frac{h}{h_0} < 0$$

Veza između relativnih i logaritamskih deformacija

$$\varphi_h = \ln\frac{h}{h_o} = \ln\frac{h_o + \Delta h}{h_o} = \ln(1+\varepsilon_h) = \ln\frac{A_0}{A}$$

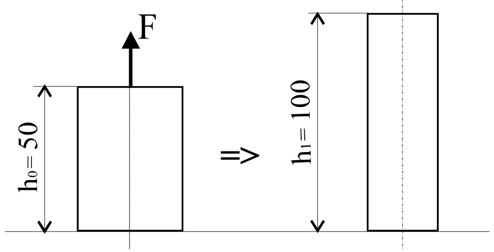
$$\varphi = \varepsilon - \frac{\varepsilon^2}{2} + \frac{\varepsilon^3}{3} - \frac{\varepsilon^4}{4} + \ldots\ldots \qquad \varphi \approx \varepsilon$$

$$V_o = V = \text{const.}$$

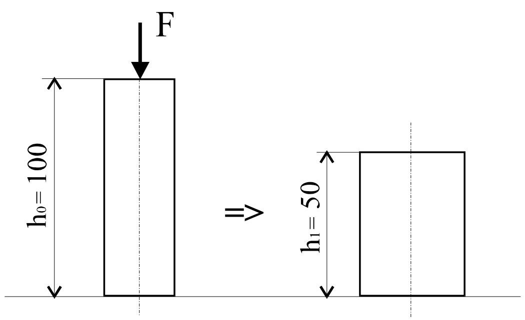
$$a_0 \cdot b_0 \cdot h_0 = a \cdot b \cdot h \Rightarrow \frac{a}{a_0}\cdot\frac{b}{b_0}\cdot\frac{h}{h_0} = 1$$

$$\boxed{\varphi_a + \varphi_b + \varphi_h = 0}$$

a) zatezanje



b) sabijanje



$$\boxed{\begin{array}{l}\Sigma\varepsilon_i \neq \varepsilon_{uk} \\ \Sigma\varphi_i = \varphi_{uk}\end{array}}$$

$$\varepsilon_{h(a)} = \frac{100}{50} - 1 = 1$$

$$\varphi_{h(a)} = \ln\frac{100}{50} = 0.693$$

$$\varepsilon_{h(a)} = \frac{50}{100} - 1 = -0.5$$

$$\varphi_{h(a)} = \ln\frac{50}{100} = -0.693$$

Homogeno i nemomogeno deformisanje

Homogeno deformisanje - deformaciono stanje u nekom trenutku u procesu isto u svim tačkama tela (npr. ravnomerno istezanje epruvete)

Pretpostavka o homogenom deformisanju

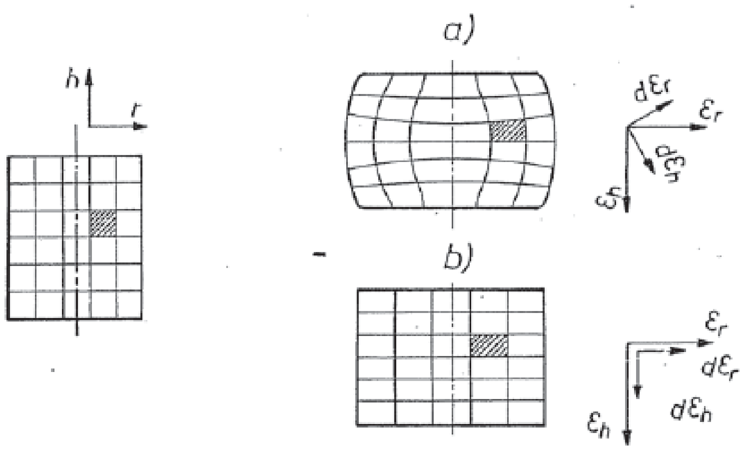
– glavni pravci deformacija za čitavo telo isti su kao i za ma koji element

– oblik tela se ne menja (paralelopiped ostaje paralelopiped)

– lokalne deformacije mogu biti znatno veće od prosečnih (problem)

– indirektan i kvantitativan pokazatelj

Nehemogeno deformisanje – uzroci: trenje, oblik (alat i obradak), mikro i makro struktura materijala obratka.



Uslovi homogenog deformisanja

— sve prave linije i ravni koje su uočene na telu (ili jednoj njegovoj zoni) ostaju takve (prave, odnosno ravni) i posle deformisanja;

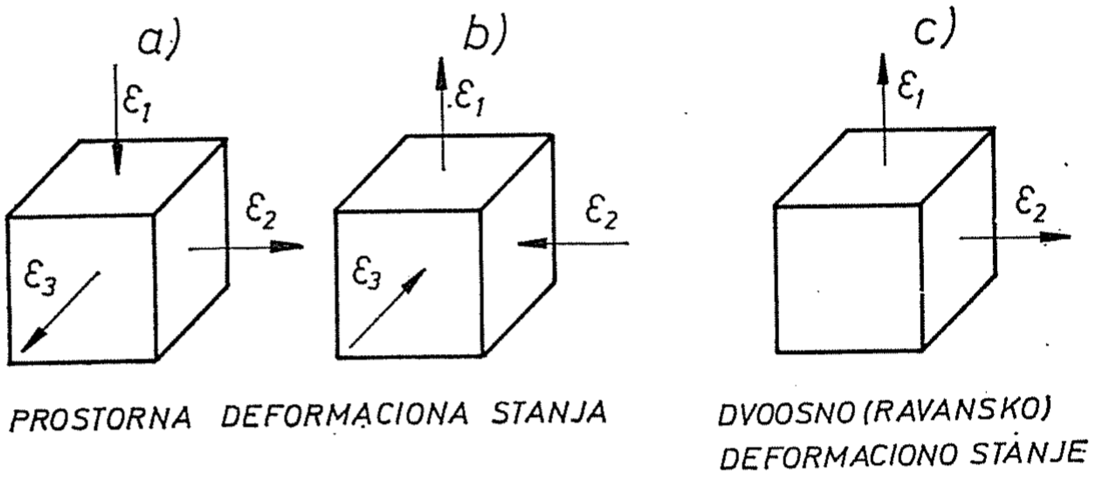
— sve međusobno paralelne linije i ravni ostaju paralelne i posle deformisanja;

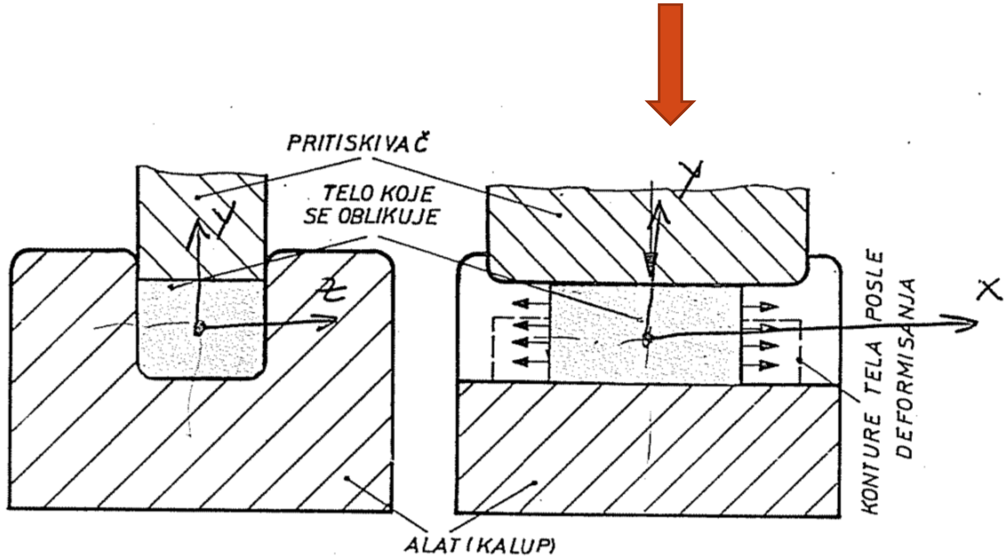
— ma koje dve međusobne paralelne dužine u okviru posmatrane zone tela menjaju se u toku deformisanja u istom odnosu.

U svakom slučaju, mera izvršenog homogenog deformisanja može biti u osnovi dvojaka: a) relativna deformacija pravolinijskih odsečaka u datim pravcima; b) promena uglova između parova pravolinijskih odsečaka koji su u početku bili međusobno upravni. Međutim, u slučaju homogenog deformisanja uvek postoje tri međusobno upravna pravca po kojima u toku procesa nastaju ekstremne linijske deformacije (najveća, najmanja i srednja) u svim tačkama posmatranog elementa ili tela. To su glavni pravci i u njima nema klizanja (ugaonih deformacija). Kako se pomenutom podelom problem nehomogenog deformisanja može svesti na problem homogenog, to znači da se u ma kom telu uvek mogu zamisliti, na primer, dovoljno male kocke, koje su tako orijentisane da se one pri deformisanju pretvaraju u pravougaone paralelopipede. To istovremeno znači da se u slučaju homogenog deformisanja deformaciono stanje može izraziti preko glavnih linijskih deformacija.

Deformaciono stanje

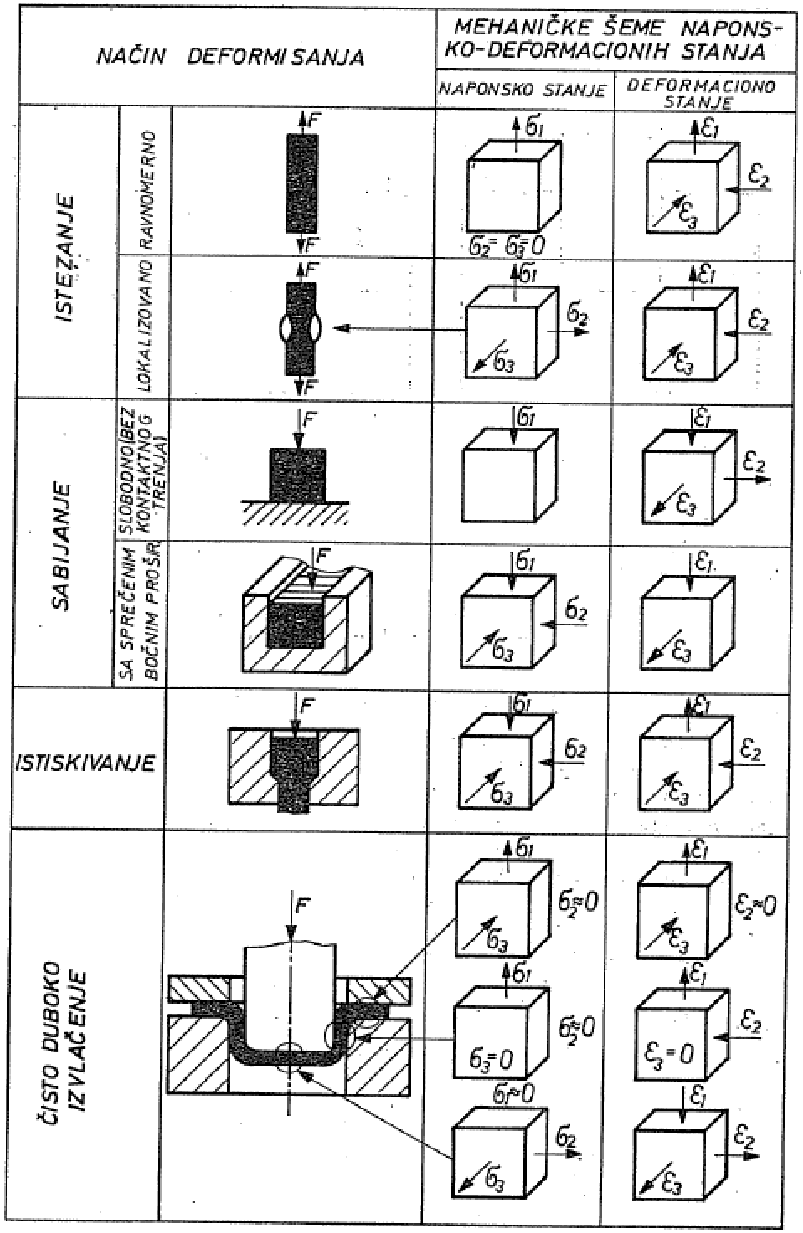
Postoje tri osnovna vida deformacionog stanja!!!





$$\sigma_z = \frac{1}{2}(\sigma_x + \sigma_y)$$

Deformaciono stanje



VAŽNO: Jednom naponskom stanju, u opštem slučaju, ne mora odgovarati jedno deformaciono stanje!!!

Male, konačne i velike deformacije

VAŽNO: U opštem slučaju, pri obradi metala deformisanjem naponsko-deformaciono stanje se menja u toku procesa, a takođe nije isto u svim tačkama tela (zavisi od koordinata)!!!

Male deformacije ($\varepsilon_{i,j}$) tj. deformacije na maloj etapi deformisanja (etapne deformacije). Ponekad se još nazivaju i priraštaj deformacija. Mala deformacija se odnosi na zapreminu malih dimenzija.

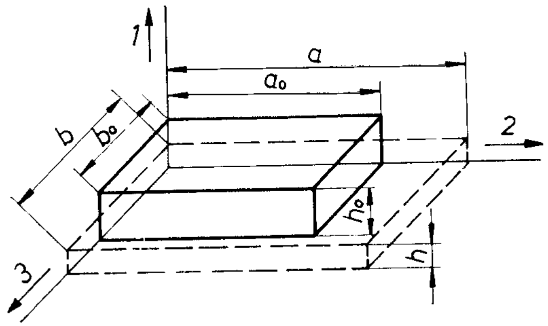
$$\varepsilon = \frac{l - l_0}{l_0} \approx \frac{l - l_0}{l} \ll 1$$

VAŽNO: Mala deformacija dovoljno malog elementa je uvek homogena!!!

Beskonačno male deformacije ($d\varepsilon_{i,j}$)

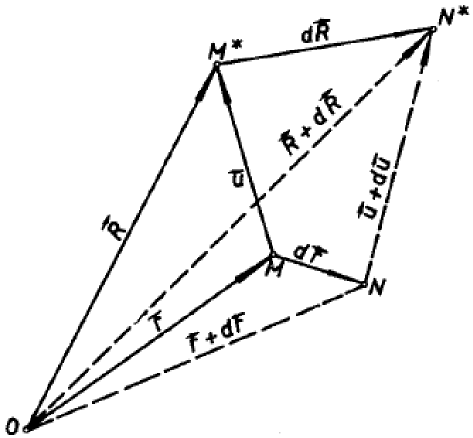
$$d\varepsilon_1 = \frac{dh}{h},\ d\varepsilon_2 = \frac{da}{a},\ d\varepsilon_3 = \frac{db}{b}$$

Velike (ukupne) deormacije (φ_{ij}) – dobijaju se sumiranjem malih (etapnih) deformacija.



Male deformacije

Pod deformacijom/deformisanjem se podrazumeva promena rastojanja između tačaka tela, a takođe i promena uglova među pravcima



VEKTOR UKUPNOG POMJERANJA

$$\vec{r} = \overrightarrow{OM} = \vec{r}(x_1,\ x_2,\ x_3)$$

Tačka M kao posljedica deformacije se pomjera u novi položaj $M^*(X_1, X_2, X_3)$

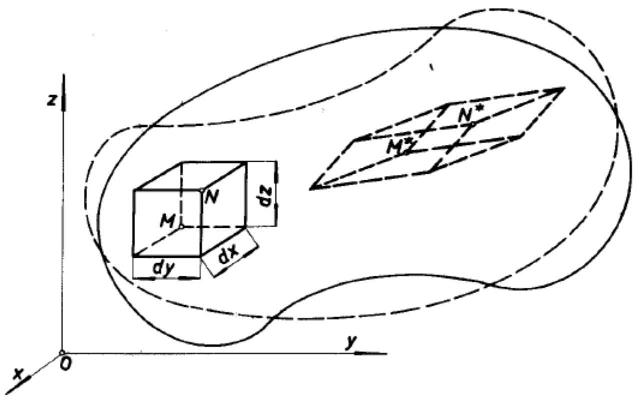
Vektor ukupnog pomjeranja je odredjen jednačinom:

$$\vec{u} = \overrightarrow{MM^*} = \vec{u}(u_1, u_2, u_3) =$$

$$= u_1\,\vec{e}_1 + u_2\,\vec{e}_2 + u_3\,\vec{e}_3 = \sum_{i=1}^{3} u_i\,\vec{e}_i$$

$$\vec{R} = \overrightarrow{OM^*} = \vec{R}(X_1, X_2, X_3) =$$

$$= X_1\,\vec{e}_1 + X_2\,\vec{e}_2 + X_3\,\vec{e}_3 = \sum_{i=1}^{3} X_i\,\vec{e}_i$$



ELEMENTARNA KOCKA DO DEFORMACIJE (pune linije) I KOSOUGLI PARALELOPIPED NAKON DEFORMACIJE (isprekidane linije)

U slučaju homogenog deformisanja, veza između koordinata tačaka pre deformacije (x_i) i nakon deformacije (X_i) je linearna.

$$X_1 = a_{11}x_1 + a_{12}x_2 + a_{13}x_3$$

$$X_2 = a_{21}x_1 + a_{22}x_2 + a_{33}x_3$$

$$X_3 = a_{31}x_1 + a_{32}x_2 + a_{33}x_3$$

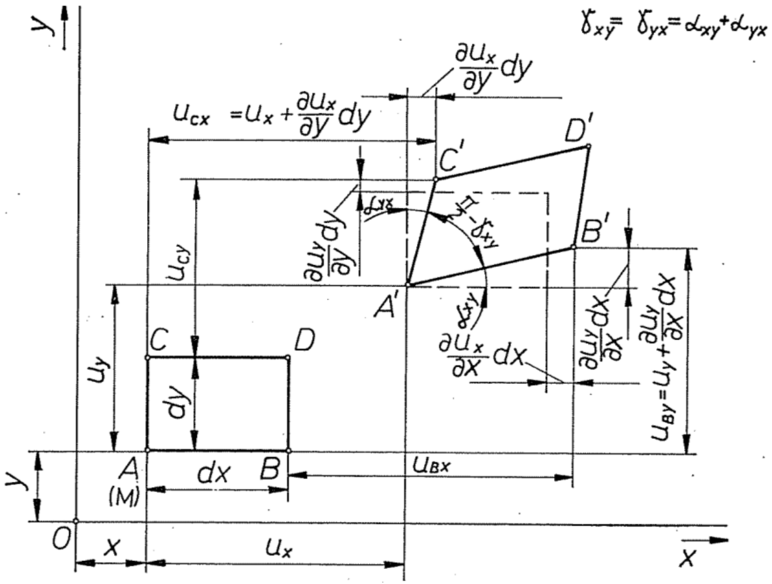
Male deformacije

Projekcije pomeranja *u* tačke na koordinatne ose:

$$x' - x = u_x$$

$$y' - y = u_y$$

$$z' - z = u_z$$

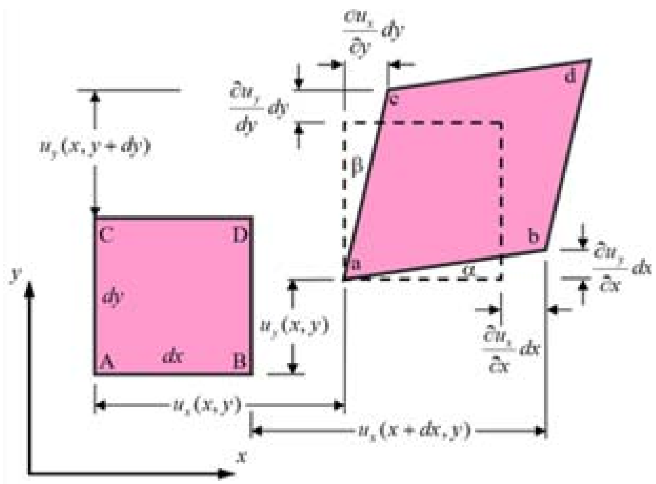


x, y, z - koordinate tačke *M* pre pomeranja
x', y', z' - koordinate tačke *M* (*M'*) nakon pomeranja

Projekcije pomeranja (u_x, u_y, u_z) su funkcije koordinata tačke M.

$$\left.\begin{aligned} u_x &= f(x, y, z) \\ u_y &= f(x, y, z) \end{aligned}\right\}\ \text{pomeranje tačke A (M)}$$

$$\left.\begin{aligned} u_{Bx} &= f(x + dx, y, z) \\ u_{Bx} &= u_x + \frac{\partial u_x}{\partial x} dx \end{aligned}\right\}\ \text{pomeranje tačke B}$$



Male deformacije

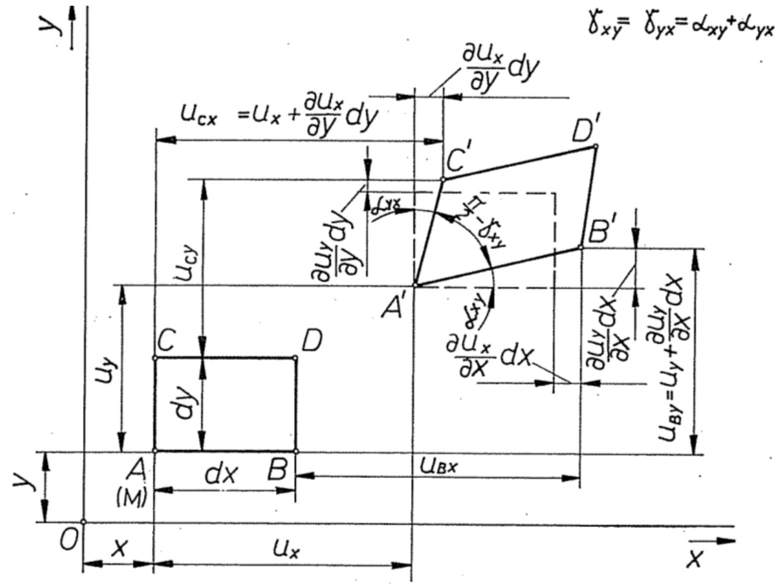
Relativna (linijska) deformacija stranice AB (dužine *dx*) i stranice AC (dužine *dy*):

$$\varepsilon_x = \frac{u_{Bx} - u_x}{dx} = \frac{u_x + \frac{\partial u_x}{\partial x}dx - u_x}{dx} = \frac{\partial u_x}{\partial x}$$

$$\varepsilon_y = \frac{u_{Cy} - u_y}{dy} = \frac{u_y + \frac{\partial u_y}{\partial y}dy - u_y}{dy} = \frac{\partial u_y}{\partial y}$$

$$u_{By} = u_y + \frac{\partial u_y}{\partial x}dx$$

$$u_{Cx} = u_x + \frac{\partial u_x}{\partial y}dy$$



Ugaone deformacije:

$\mathrm{tg}\,\alpha_{xy} \approx \alpha_{xy}$; $\mathrm{tg}\,\alpha_{yx} \approx \alpha_{yx}$ - za male uglove

$$\alpha_{xy} = \frac{u_{By} - u_y}{u_{Bx} + dx - u_x} = \frac{u_y + \frac{\partial u_y}{\partial x}dx - u_y}{u_x + \frac{\partial u_x}{\partial x}dx + dx - u_x} = \frac{\frac{\partial u_y}{\partial x}}{1 + \frac{\partial u_x}{\partial x}}$$

$$\frac{\partial u_x}{\partial x} = \varepsilon_x << 1$$

$$\alpha_{xy} \approx \frac{\partial u_y}{\partial x}$$

$$\alpha_{yx} \approx \frac{\partial u_x}{\partial y}$$

Ugao klizanja

$$\boxed{\gamma_{xy} = \alpha_{xy} + \alpha_{yx} = \frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y}}$$

Male deformacije

Relativne linijske (male) deformacija (ε) :

$$\varepsilon_x = \frac{\partial u_x}{\partial x}$$

$$\varepsilon_y = \frac{\partial u_y}{\partial y}$$

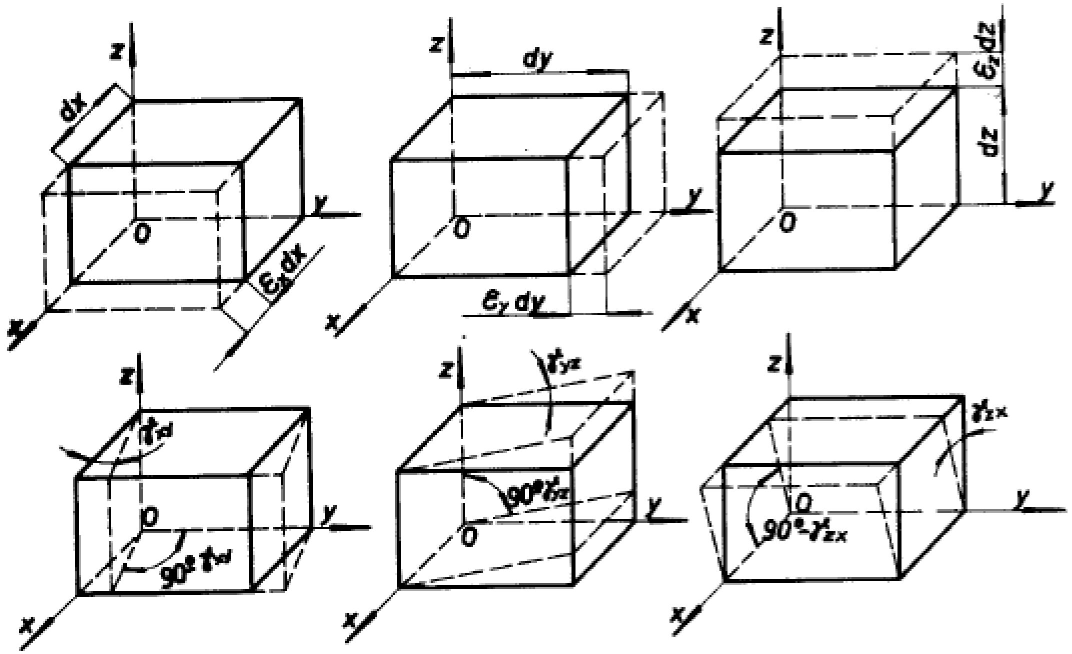
$$\varepsilon_z = \frac{\partial u_z}{\partial z}$$

Relative ugaone (male) deformacije (klizanje - γ):

$$\gamma_{xy} = \gamma_{yx} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}$$

$$\gamma_{yz} = \gamma_{zy} = \frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}$$

$$\gamma_{xz} = \gamma_{zx} = \frac{\partial u_z}{\partial z} + \frac{\partial u_x}{\partial x}$$



GEOMETRIJSKO ZNAČENJE KOMPONENATA
MALE DEFORMACIJE

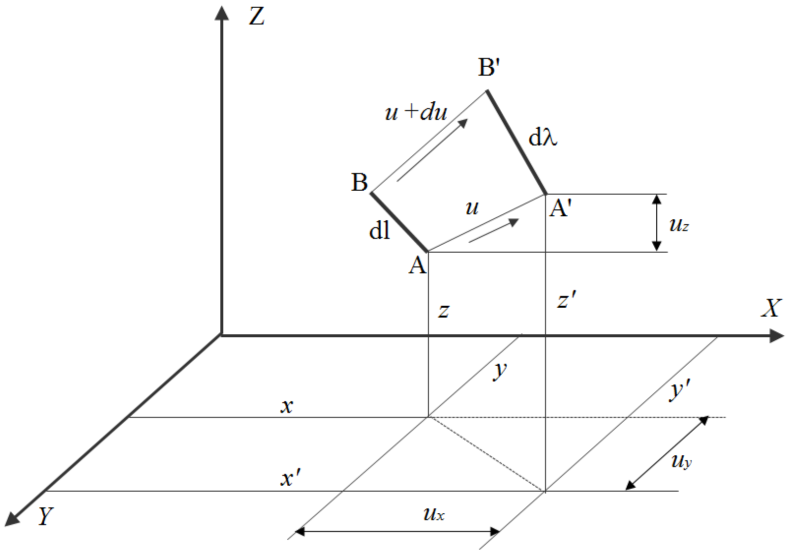
Konačne deformacije

Konačne deformacije (e) – u slučaju većih pomeranja ne mogu se zanemariti kvadrati i međusobni proizvodi malih članova, koji utiču na

$$\varepsilon = \frac{d\lambda - dl}{dl}$$

dλ (M'N') –dužina elementa nakon deformacije
dl (MN)– dužina elementa pre deformacije

$$d\lambda = (1 + \varepsilon)\, dl$$



$$x' = x + u_x(x, y, z)$$
$$y' = y + u_y(x, y, z)$$
$$z' = z + u_z(x, y, z)$$

$$dx' = \left(1 + \frac{\partial u_x}{\partial x}\right) dx + \frac{\partial u_x}{\partial y} dy + \frac{\partial u_x}{\partial z} dz$$

$$dy' = \frac{\partial u_y}{\partial x} dx + \left(1 + \frac{\partial u_y}{\partial y}\right) dy + \frac{\partial u_y}{\partial z} dz$$

$$dz' = \frac{\partial u_z}{\partial x} dx + \frac{\partial u_z}{\partial y} dy + \left(1 + \frac{\partial u_z}{\partial z}\right) dz$$

$$dl^2 = dx^2 + dy^2 + dz^2$$
$$d\lambda^2 = dx'^2 + dy'^2 + dz'^2$$

$$d\lambda^2 = dl^2 + 2e_{xx}\, dx^2 + 2e_{yy}\, dy^2 + 2e_{zz}\, dz^2 +$$
$$2e_{xy}\, dx\, dy + 2e_{yz}\, dy\, dx + 2e_{zx}\, dz\, dx$$

Konačne deformacije

Komponenennte konačne deformacije (e_{ij}):

$$e_{xx} = \frac{\partial u_x}{\partial x} + \frac{1}{2}\left[\left(\frac{\partial u_x}{\partial x}\right)^2 + \left(\frac{\partial u_y}{\partial x}\right)^2 + \left(\frac{\partial u_z}{\partial x}\right)^2\right]$$

$$e_{yy} = \frac{\partial u_y}{\partial y} + \frac{1}{2}\left[\left(\frac{\partial u_x}{\partial y}\right)^2 + \left(\frac{\partial u_y}{\partial y}\right)^2 + \left(\frac{\partial u_z}{\partial y}\right)^2\right]$$

$$e_{zz} = \frac{\partial u_z}{\partial z} + \frac{1}{2}\left[\left(\frac{\partial u_x}{\partial z}\right)^2 + \left(\frac{\partial u_y}{\partial z}\right)^2 + \left(\frac{\partial u_z}{\partial z}\right)^2\right]$$

$$e_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial x}\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial x}\frac{\partial u_z}{\partial y}$$

$$e_{yz} = \frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y} + \frac{\partial u_x}{\partial y}\frac{\partial u_x}{\partial z} + \frac{\partial u_y}{\partial y}\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\frac{\partial u_z}{\partial z}$$

$$e_{zx} = \frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z} + \frac{\partial u_x}{\partial z}\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial z}\frac{\partial u_y}{\partial x} + \frac{\partial u_z}{\partial z}\frac{\partial u_z}{\partial x}$$

Tenzor konačne deformacije:

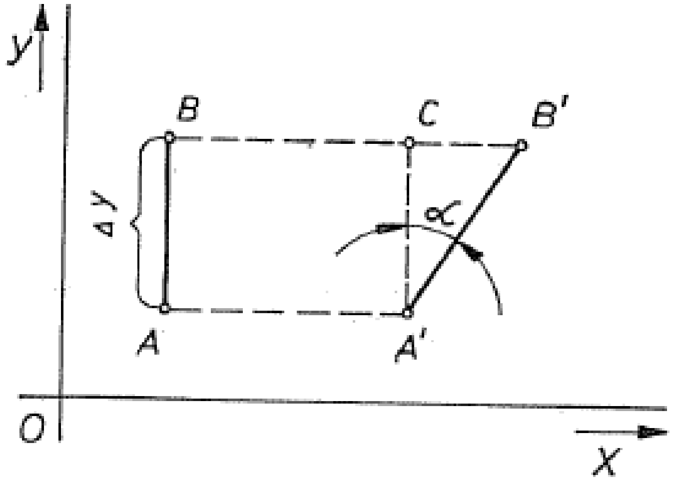
$$\boxed{T_e = \begin{Bmatrix} e_{xx} & e_{xy} & e_{xz} \\ e_{yx} & e_{yy} & e_{yz} \\ e_{zx} & e_{zy} & e_{zz} \end{Bmatrix}}$$

$$\left.\begin{aligned} \frac{d\lambda}{dl} &= \frac{d\lambda}{dx} = 1 + \varepsilon_{xx} \\ \frac{d\lambda}{dl} &= \frac{d\lambda}{dx} = \sqrt{2e_{xx} + 1} \end{aligned}\right\} \quad \boxed{\varepsilon_{xx} = \sqrt{2e_{xx} + 1} - 1 \approx e_{xx}}$$

Veza malih i konačnih deformacija

Komponenennte konačne deformacije (e_{ij}):

$$\left.\begin{aligned} \operatorname{tg}\alpha &= \frac{\Delta u_x}{\Delta y} \\ \Delta y &\to 0 \end{aligned}\right\} \quad \frac{\partial u_x}{\partial y}$$



$$\varepsilon_{xx} = \sqrt{2e_{xx} + 1} - 1 \approx e_{xx}$$

$$\varepsilon_{xx} = \sqrt{\left(1 + \frac{\partial u_x}{\partial x}\right)^2 + \left(\frac{\partial u_y}{\partial x}\right)^2 + \left(\frac{\partial u_z}{\partial x}\right)^2} - 1 \approx e_{xx}$$

$$\boxed{\begin{aligned} e_{xx} &\approx \varepsilon_{xx} = \varepsilon_x = \frac{\partial u_x}{\partial x} \\ e_{xy} &\approx \gamma_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \end{aligned}}$$

Male, konačne i velike deformacije

- Iz izraza za komponente konačne deformacije mogu se dobiti male deformacije zanemarivanjem članova višeg reda (linearicazija).
- Ako su deformacije znatne onda se tačne deformacije mogu dobiti samo na osnovu punih izraza za komponente konačne deformacije (kvadratni članovi i međusobni proizvodi se ne mogu zanemariti jer su u pitanju znatne deformacije)
- Zbog matematičkih poteškoća u teoriji obrade deformisanjem se koriste male deformacije (teorija priraštaja beskonačno malih deformacija)!!!

Homogene deformacije

➢ **Velika (ukupna, zbirna**) deformacija dobija se kao zbir priraštaja malih deformacija

$$\frac{d(\Delta a)}{\Delta a}\ ;\ \frac{d(\Delta b)}{\Delta b}\ ;\ \frac{d(\Delta c)}{\Delta c}$$

$$\varphi_a = ln\frac{\Delta a_2}{\Delta a_1}\ ;\ \varphi_b = ln\frac{\Delta b_2}{\Delta b_1}\ ;\ \varphi_c = ln\frac{\Delta c_2}{\Delta c_1}$$

$$\varphi_a = ln\frac{a_2}{a_1}\ ;\ \varphi_b = ln\frac{b_2}{b_1}\ ;\ \varphi_c = ln\frac{c_2}{c_1}$$

Tenzor (male) deformacije

- Deformaciono stanje u okolini tačke u potpunosti je definisano tenzorom deformacija i neprekidna je funkcija koordinata

$$T_\varepsilon = \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{yx} & \varepsilon_y & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{zx} & \frac{1}{2}\gamma_{zy} & \varepsilon_z \end{bmatrix}$$

*) Tenzorske veličine, u opštem slučaju, mogu biti: male deformacije (ε_x ,...), beskonačno male deformacije—priraštaji ($d\varepsilon_x$,...) i konačne deformacije (e_{xx},...), a kako će se kasnije videti, takođe i brzine deformacija.

- Tenzor deformacija izražen preko glavnih deformacija

$$T_\varepsilon = \begin{bmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ 0 & 0 & \varepsilon_3 \end{bmatrix}$$

Devijator tenzora deformacija. Invarijante

- Srednja linijska deformacija

$$\varepsilon_m = \frac{\varepsilon_x + \varepsilon_y + \varepsilon_z}{3} = 0$$

- Sferni tenzor plastične deformacije $T_\varepsilon(s)$ – izražava promenu zapremine

$$T_\varepsilon(s) = \begin{bmatrix} \varepsilon_m & 0 & 0 \\ 0 & \varepsilon_m & 0 \\ 0 & 0 & \varepsilon_m \end{bmatrix} = 0$$

- Razlaganje tenzora deformacije na sferični $T_\varepsilon(s)$ i devijatorski deo D_ε

$$T_\varepsilon = T_{s(\varepsilon)} + D_\varepsilon = D_\varepsilon$$

- Devijator tenzora deformacije D_ε – izražava promenu oblika

$$D_\varepsilon = T_\varepsilon = \begin{bmatrix} \varepsilon_x - \varepsilon_m & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{yx} & \varepsilon_y - \varepsilon_m & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{zx} & \frac{1}{2}\gamma_{zy} & \varepsilon_z - \varepsilon_m \end{bmatrix} = \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{yx} & \varepsilon_y & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{zx} & \frac{1}{2}\gamma_{zy} & \varepsilon_z \end{bmatrix}$$

Devijator tenzora deformacija. Invarijante

- Određivanje glavnih (linijskih) deformacija

$$\begin{vmatrix} \varepsilon_x - \varepsilon & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{yx} & \varepsilon_y - \varepsilon & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{zx} & \frac{1}{2}\gamma_{zy} & \varepsilon_z - \varepsilon \end{vmatrix} = 0$$

$$\varepsilon^3 - J_1\varepsilon^2 + J_2\varepsilon - J_3 = 0$$

- Invarijante tenzora deformacije

$$J_1(T_\varepsilon) = J_1(D_\varepsilon) \overset{*)}{\equiv} \varepsilon_x + \varepsilon_y + \varepsilon_z = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0$$

$$J_2(T_\varepsilon) \neq J_2(D_\varepsilon) \equiv \varepsilon_x\varepsilon_y + \varepsilon_y\varepsilon_z + \varepsilon_z\varepsilon_x - \frac{1}{4}\left(\gamma_{xy}^2 + \gamma_{yz}^2 + \gamma_{zx}^2\right) = \varepsilon_1\varepsilon_2 + \varepsilon_2\varepsilon_3 + \varepsilon_3\varepsilon_1 = \text{const.}$$

$$J_3(T_\varepsilon) = J_3(D_\varepsilon) \equiv \begin{vmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{yx} & \varepsilon_y & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{zx} & \frac{1}{2}\gamma_{zy} & \varepsilon_z \end{vmatrix} =$$

$$= \varepsilon_x\varepsilon_y\varepsilon_z + \frac{1}{4}\gamma_{xy}\gamma_{yz}\gamma_{xz} - \frac{1}{4}\left(\varepsilon_x\gamma_{yz}^2 + \varepsilon_y\gamma_{zx}^2 + \varepsilon_z\gamma_{xy}^2\right) = \varepsilon_1\varepsilon_2\varepsilon_3 = \text{const.}$$

Efektivna (ekvivalentna) deformacija. Ugaona deformacija

- Efektivna(ekvivalentna) deformacija (ε_e)

- jednoosno zatezanje/sabijanje

$$\varepsilon_e = \frac{2}{\sqrt{3}}\sqrt{J_2}$$

$$\boxed{\varepsilon_e = \varepsilon_{\max}}$$

$$\varepsilon_e = \frac{\sqrt{2}}{3}\sqrt{(\varepsilon_x-\varepsilon_y)^2+(\varepsilon_y-\varepsilon_z)^2+(\varepsilon_z-\varepsilon_x)^2+\frac{3}{2}\left(\gamma_{xy}^2+\gamma_{yz}^2+\gamma_{zx}^2\right)} =$$

$$= \frac{\sqrt{2}}{3}\sqrt{(\varepsilon_1-\varepsilon_2)^2+(\varepsilon_2-\varepsilon_3)^2+(\varepsilon_3-\varepsilon_1)^2} = \sqrt{\frac{2}{3}\left(\varepsilon_1^2+\varepsilon_2^2+\varepsilon_3^2\right)}$$

- Uopštena ugaona deformacija (γ_e)

$$\gamma_e = 2\sqrt{|J_2(T_\varepsilon)|} =$$

$$= \sqrt{\frac{2}{3}}\sqrt{(\varepsilon_x-\varepsilon_y)^2+(\varepsilon_y-\varepsilon_z)^2+(\varepsilon_z-\varepsilon_x)^2+\frac{3}{2}\left(\gamma_{xy}^2+\gamma_{zy}^2+\gamma_{zx}^2\right)} =$$

$$= \sqrt{\frac{2}{3}}\sqrt{(\varepsilon_1-\varepsilon_2)^2+(\varepsilon_2-\varepsilon_3)^2+(\varepsilon_3-\varepsilon_1)^2}$$

$$\varepsilon_x\varepsilon_y+\varepsilon_y\varepsilon_z+\varepsilon_z\varepsilon_x = -\frac{1}{6}\left[(\varepsilon_x-\varepsilon_y)^2+(\varepsilon_y-\varepsilon_z)^2+(\varepsilon_z-\varepsilon_x)^2\right]$$

Oktaedarska deformacija

$$\gamma_0 = \frac{2}{3}\sqrt{(\varepsilon_x - \varepsilon_y)^2 + (\varepsilon_y - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_x)^2 + \frac{3}{2}\left(\gamma_{xy}^2 + \gamma_{yz}^2 + \gamma_{zx}^2\right)} =$$

$$= \frac{2}{3}\sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2}$$

$$\varepsilon_0 = \varepsilon_m = \frac{\varepsilon_1 + \varepsilon_2 + \varepsilon_3}{3} = 0$$

U ravnima koje prolaze kroz jednu glavnu koordinatnu osu, a sa druge dve zaklapaju ugao od 45°, pojavljuju se ekstremne, tj. glavne ugaone deformacije (klizanja). Tačnije rečeno, glavna ugaona deformacija predstavlja promenu pravog ugla koji obrazuju ravni glavnih smičućih napona.

- Veza između glavnih ugaonih i linijskih deformacija

$$\gamma_{12} = 2\,\frac{1 - \dfrac{1+\varepsilon_1}{1+\varepsilon_2}}{1 + \dfrac{1+\varepsilon_1}{1+\varepsilon_2}} = 2\,\frac{\varepsilon_2 - \varepsilon_1}{2 + \varepsilon_1 + \varepsilon_2} = \frac{\varepsilon_2 - \varepsilon_1}{1 + \dfrac{\varepsilon_1 + \varepsilon_2}{2}}$$

$$\gamma_{12} = \varepsilon_2 - \varepsilon_1 = \gamma_{21}$$

$$\gamma_{23} = \varepsilon_3 - \varepsilon_2$$

$$\gamma_{31} = \varepsilon_1 - \varepsilon_3$$

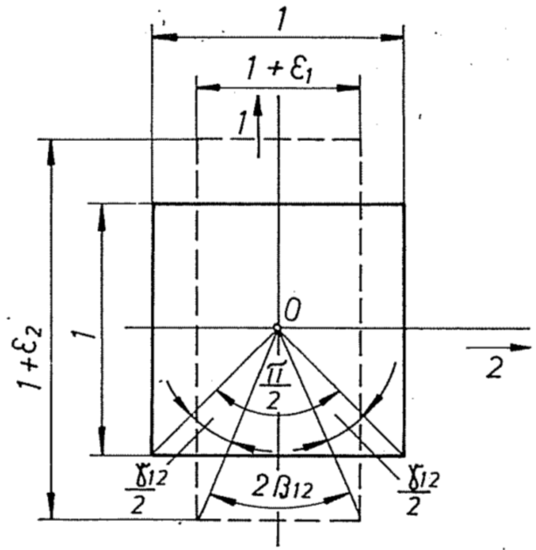
$$\boxed{\gamma_{12} + \gamma_{23} + \gamma_{31} = 0}$$

$$\gamma_e = \sqrt{3}\,\varepsilon_e$$

$$\gamma_0 = (0{,}816 \div 0{,}941)\,|\gamma|_{max}$$

$$\gamma_e = (1 \div 1{,}155)\,|\gamma|_{max}$$

$$\varepsilon_e = (1 \div 1{,}155)\,|\varepsilon|_{max}$$



$$\frac{\gamma_{12}}{2} = \mathrm{tg}\left(\frac{\pi}{4} - \beta_{12}\right) = \frac{1 - \mathrm{tg}\,\beta_{12}}{1 + \mathrm{tg}\,\beta_{12}}$$

$$\mathrm{tg}\,\beta_{12} = \frac{1 + \varepsilon_1}{1 + \varepsilon_2}$$

Morh-ov krug deformacija. Koeficijent deformacije

- Poluprečnici krugova

$$\frac{1}{2}(\varepsilon_1 - \varepsilon_3)\,; \quad \frac{1}{2}(\varepsilon_1 - \varepsilon_2)\,; \quad \frac{1}{2}(\varepsilon_2 - \varepsilon_3)$$

- Koeficijent deformacije – pokazatelj deformacionog stanja

$$\nu_\varepsilon = \frac{\overline{0_1 B}}{\overline{0_1 C}} \qquad \operatorname{tg} \varphi_\varepsilon = \frac{\nu_\varepsilon}{\sqrt{3}}$$

$$\nu_\varepsilon = \frac{\varepsilon_2 - \dfrac{\varepsilon_1 + \varepsilon_3}{2}}{\dfrac{\varepsilon_1 - \varepsilon_3}{2}} = \frac{2\varepsilon_2 - \varepsilon_1 - \varepsilon_3}{\varepsilon_1 - \varepsilon_3}$$

$$\varepsilon_2 = -(\varepsilon_1 + \varepsilon_3)$$

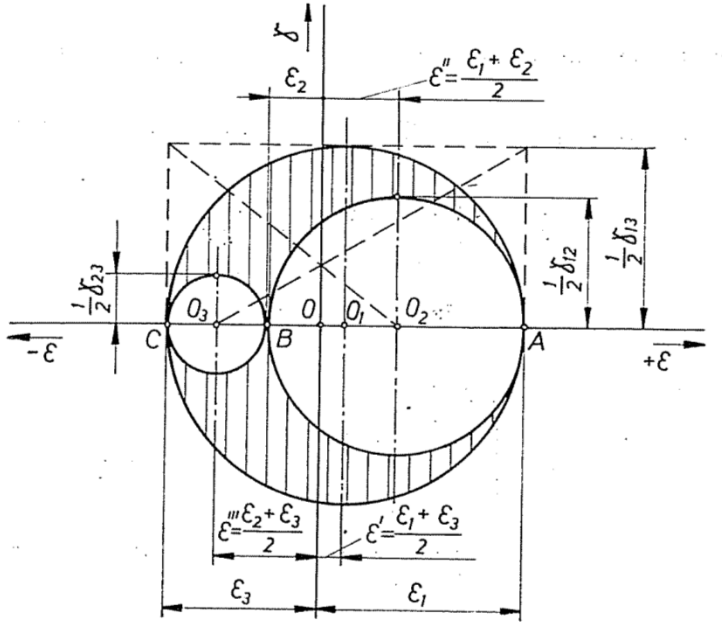
$$\nu_\varepsilon = -3\,\frac{\varepsilon_1 + \varepsilon_3}{\varepsilon_1 - \varepsilon_3}$$

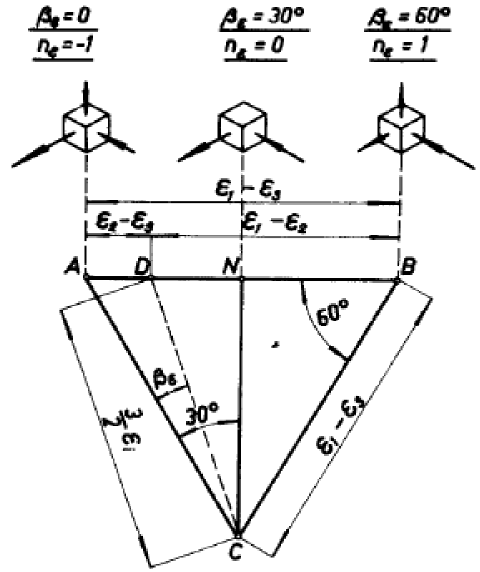
za $\varepsilon_1 = \varepsilon_2 = -\frac{1}{2}\varepsilon_3$: $\nu_\varepsilon = 1$

za $\varepsilon_2 = 0\,;\ \varepsilon_1 = -\varepsilon_3$: $\nu_\varepsilon = 0$

za $-\varepsilon_1 = -\varepsilon_2 = \frac{1}{2}\varepsilon_3$: $\nu_\varepsilon = 1$

Napominje se da je u oblasti elastičnosti uvek $\boxed{\nu_\sigma = \nu_\varepsilon}$, ali u oblasti plastičnosti to ne mora biti slučaj.





MEHANIČKA ŠEMA DEFORMACIJA

Pokazatelj deformacije

- Velike deformacije, homogemo deformisanje → koristi se teorija malih deformacija

$$\varphi_1 = -(\varphi_2 + \varphi_3)$$

$$|\varphi_1| = |\varphi|_{max}$$

$$\varphi_e = \sqrt{\frac{2}{3}\left(\varphi_1^2 + \varphi_2^2 + \varphi_3^2\right)}$$

- Pokazatelj deformacije C_φ

$$C_\varphi = \frac{\varphi_e}{|\varphi|_{max}}$$

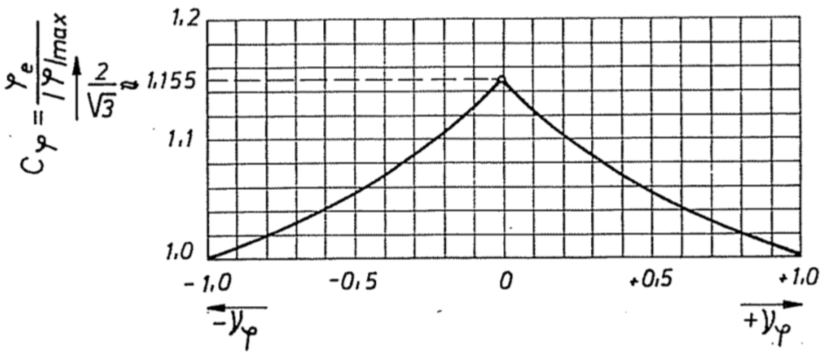
za $\varphi_2 = \varphi_1$ biće $\varphi_e = -\varphi_3 = |\varphi|_{max}$ i $C_\varphi = 1$

za $\varphi_2 = \varphi_3$ biće $\varphi_e = \varphi_1 = |\varphi|_{max}$ i $C_\varphi = 1$

za $\varphi_2 = 0$ tj. $\varphi_1 = -\varphi_3$ biće

$$\varphi_e = \frac{2}{\sqrt{3}}\varphi_1 = \frac{2}{\sqrt{3}}|\varphi_3| \quad \text{i} \quad C_\varphi = \frac{2}{\sqrt{3}} \approx 1{,}15$$

$$\boxed{1 \leqslant C_\varphi \leqslant \frac{2}{\sqrt{3}} \approx 1{,}15}$$



Kompatibilnost deformacija (Saint-Venant-ovi uslovi)

Komponente deformacije su definisane komponentama pomeranja (neprekidne funkcije koordinata) → komponente deformacije ne mogu imati proizvolje, međusobno nezavisne vrednosti!!!!

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \, \partial y}$$

$$\frac{\partial^2 \varepsilon_y}{\partial z^2} + \frac{\partial^2 \varepsilon_z}{\partial y^2} = \frac{\partial^2 \gamma_{yz}}{\partial y \, \partial z}$$

$$\frac{\partial^2 \varepsilon_z}{\partial x^2} + \frac{\partial^2 \varepsilon_x}{\partial z^2} = \frac{\partial^2 \gamma_{zx}}{\partial z \, \partial x}$$

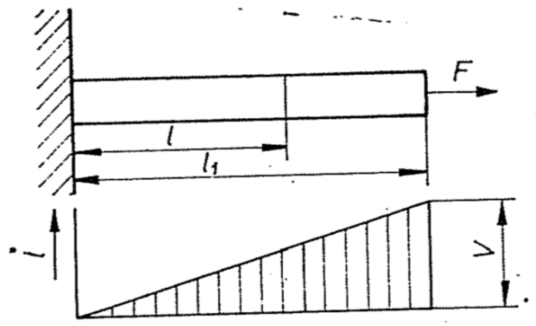
$$\frac{\partial^2 \varepsilon_x}{\partial y \, \partial z} = \frac{1}{2} \frac{\partial}{\partial x} \left(- \frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{zx}}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z} \right)$$

$$\frac{\partial^2 \varepsilon_y}{\partial z \, \partial x} = \frac{1}{2} \frac{\partial}{\partial y} \left(\frac{\partial \gamma_{yz}}{\partial x} - \frac{\partial \gamma_{zx}}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z} \right)$$

$$\frac{\partial^2 \varepsilon_z}{\partial x \, \partial y} = \frac{1}{2} \frac{\partial}{\partial z} \left(\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{zx}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z} \right)$$

Brzina deformacije. Brzina deformisanja

$$\dot{\varphi}_x = \frac{d\varphi_x}{dt} = \frac{1}{l}\frac{dl}{dt} = \frac{\dot{l}}{l}\left[sec^{-1}\right]$$



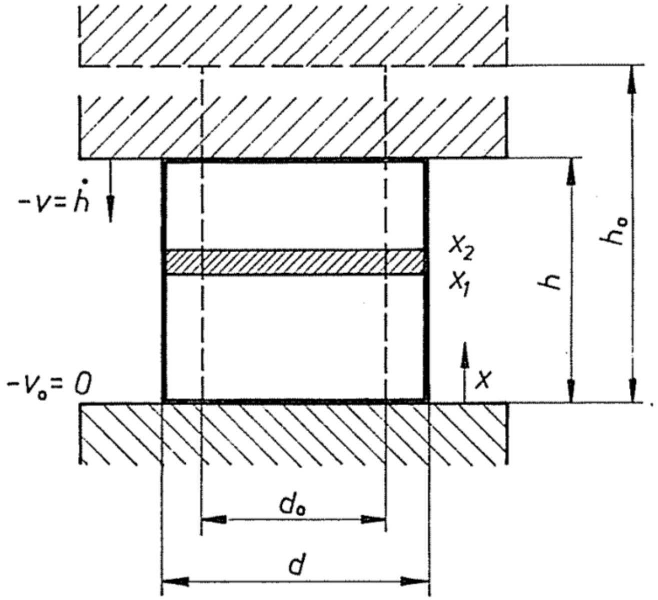
V– BRZINA RADNOG ORGANA MAŠINE KOJI VRŠI ISTEZANJE

$$\dot{\varphi} = -\frac{v}{h} = \frac{\dot{h}}{h}$$

$$\frac{v_x}{x} = \frac{v_h}{h}$$

$$\varphi = \int_{t_0}^{t}\dot{\varphi}dt = -\int_{t_0}^{t}\frac{v}{h}\,dt = \int_{t_0}^{t}\frac{\dot{h}dt}{h} = \int_{h_0}^{h}\frac{dh}{h} = ln\,\frac{h}{h_0}$$

$$\bar{v} = \frac{\Delta l}{\Delta t} = \frac{l - l_0}{\Delta t}$$ - srednja brzina deformisanja



Brzina deformisanja i brzina deformacije

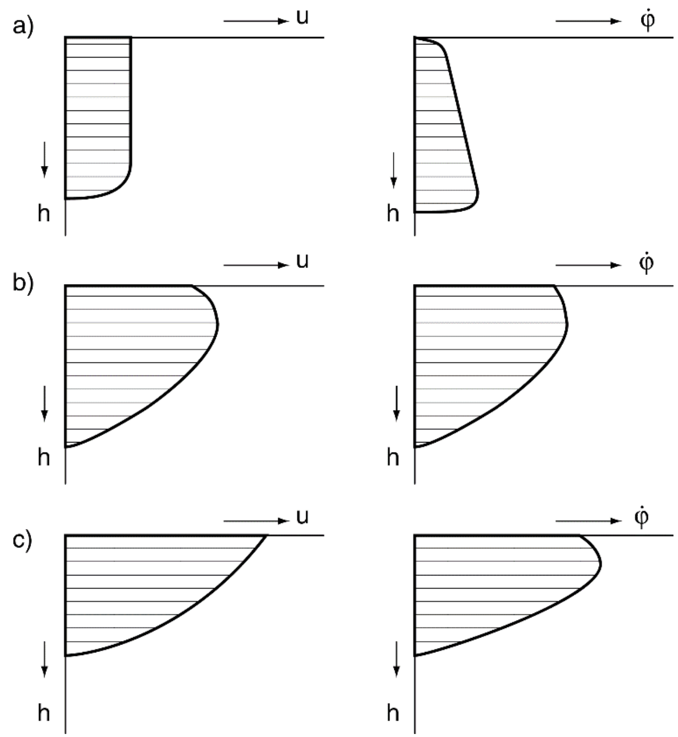
- Brzina deformisanja *u* (brzina pritiskivača mašine)

$$u = \frac{ds}{dt} \approx \frac{\Delta h}{\Delta t} = \frac{h_0 - h_1}{t_0 - t_1} \left[m/s\right]$$

-Brzina deformacije
(brzina relativnog pomeranja čestica materijala)

$$\dot{\varphi} = \frac{d\varphi}{dt} = \frac{dh/h}{dt} = \frac{1}{h}\frac{dh}{dt} = \frac{1}{h} \cdot u \left[s^{-1}\right]$$

$$\dot{\varphi}_{sr} = \frac{\varphi}{t_d} = \frac{\ln\frac{h_0}{h}}{t_d}$$



Vrsta mašine	Brzina deformisanja u [mm/s]	Brzina deformacije $\dot{\varphi}\,[s^{-1}]$
Hidraulične prese	30 ÷ 500	0,01 ÷ 10
Mehaničke prese	400 ÷ 600	4 ÷ 25
Čekići	5000 ÷ 7000	40 ÷ 160

Brzina deformacije

Komponente brzine (linijskog) pomeranja materijalne tačke (neprekidne funkcije)

$$\dot{u}_x = f_x(x, y, z, t) \qquad \dot{u}_x = \frac{\partial u_x}{\partial t}$$

$$\dot{u}_y = f_y(x, y, z, t) \qquad \dot{u}_y = \frac{\partial u_y}{\partial t}$$

$$\dot{u}_z = f_z(x, y, z, t) \qquad \dot{u}_z = \frac{\partial u_z}{\partial t}$$

Komponente brzine deformacije

$$\dot{\varepsilon}_x = \frac{\partial \dot{u}_x}{\partial x}$$

$$\dot{\varepsilon}_x = \frac{\partial u_x}{\partial x\, \partial t} = \frac{\partial}{\partial t}\left(\frac{\partial u_x}{\partial x}\right) = \frac{\partial \varepsilon_x}{\partial t}$$

$$\dot{\gamma}_{xy} = \frac{\partial \dot{u}_x}{\partial y} + \frac{\partial \dot{u}_y}{\partial x}$$

$$\dot{\gamma}_{xy} = \frac{\partial u_x}{\partial y\, \partial t} + \frac{\partial u_y}{\partial x\, \partial t} = \frac{\partial}{\partial t}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right) = \frac{\partial \gamma_{xy}}{\partial t}$$

Brzina deformacije

a) izvod brzine pomeranja po koordinatama

b) promena deformacije u jedinici vremena

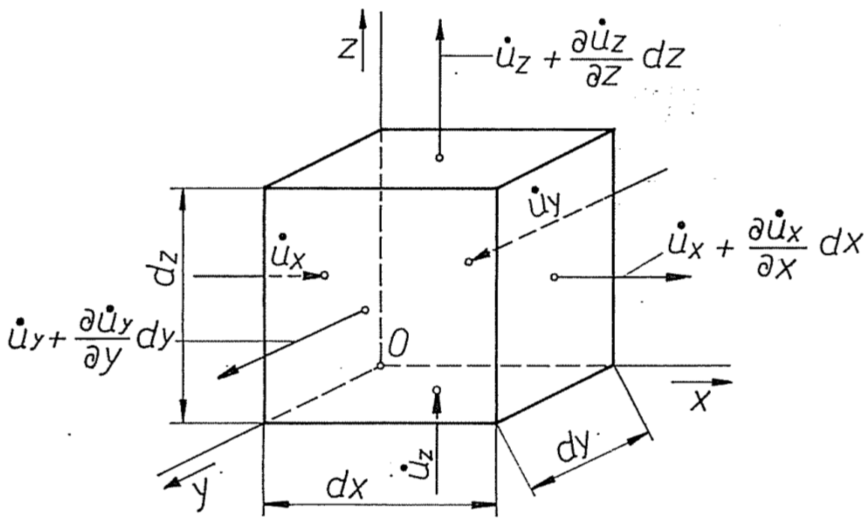
$$\dot{\varepsilon}_x = \frac{\partial \dot{u}_x}{\partial x} = \frac{\partial \varepsilon_x}{\partial t} \qquad \dot{\gamma}_{xy} = \frac{\partial \dot{u}_x}{\partial y} + \frac{\partial \dot{u}_y}{\partial x} = \frac{\partial \gamma_{xy}}{\partial t}$$

$$\dot{\varepsilon}_y = \frac{\partial \dot{u}_y}{\partial y} = \frac{\partial \varepsilon_y}{\partial t} \qquad \dot{\gamma}_{yz} = \frac{\partial \dot{u}_y}{\partial z} + \frac{\partial \dot{u}_z}{\partial y} = \frac{\partial \gamma_{yz}}{\partial t}$$

$$\dot{\varepsilon}_z = \frac{\partial \dot{u}_z}{\partial z} = \frac{\partial \varepsilon_z}{\partial t} \qquad \dot{\gamma}_{zx} = \frac{\partial \dot{u}_z}{\partial x} + \frac{\partial \dot{u}_x}{\partial z} = \frac{\partial \gamma_{zx}}{\partial t}$$

Tenzor brzine deformacije

$$T_{\dot{\varepsilon}} = \begin{Bmatrix} \dot{\varepsilon}_x & \frac{1}{2}\dot{\gamma}_{xy} & \frac{1}{2}\dot{\gamma}_{xz} \\ \frac{1}{2}\dot{\gamma}_{yx} & \dot{\varepsilon}_y & \frac{1}{2}\dot{\gamma}_{yz} \\ \frac{1}{2}\dot{\gamma}_{zx} & \frac{1}{2}\dot{\gamma}_{zy} & \dot{\varepsilon}_z \end{Bmatrix}$$



Jednačina kontinuiteta

$$\dot{\varepsilon}_x + \dot{\varepsilon}_y + \dot{\varepsilon}_z = \frac{\partial \dot{u}_x}{\partial x} + \frac{\partial \dot{u}_y}{\partial y} + \frac{\partial \dot{u}_z}{\partial z} = 0$$

$$\dot{\varepsilon}_m = \frac{\dot{\varepsilon}_x + \dot{\varepsilon}_y + \dot{\varepsilon}_z}{3} = \frac{\dot{\varepsilon}_1 + \dot{\varepsilon}_2 + \dot{\varepsilon}_3}{3} = 0$$

$$D_{\dot{\varepsilon}} = \begin{Bmatrix} \dot{\varepsilon}_x - \dot{\varepsilon}_m & \frac{1}{2}\dot{\gamma}_{xy} & \frac{1}{2}\dot{\gamma}_{xz} \\ \frac{1}{2}\dot{\gamma}_{yx} & \dot{\varepsilon}_y - \dot{\varepsilon}_m & \frac{1}{2}\dot{\gamma}_{yz} \\ \frac{1}{2}\dot{\gamma}_{zx} & \frac{1}{2}\dot{\gamma}_{zy} & \dot{\varepsilon}_z - \dot{\varepsilon}_m \end{Bmatrix} = \begin{Bmatrix} \dot{\varepsilon}_x & \frac{1}{2}\dot{\gamma}_{xy} & \frac{1}{2}\dot{\gamma}_{xz} \\ \frac{1}{2}\dot{\gamma}_{yx} & \dot{\varepsilon}_y & \frac{1}{2}\dot{\gamma}_{yz} \\ \frac{1}{2}\dot{\gamma}_{zx} & \frac{1}{2}\dot{\gamma}_{zy} & \dot{\varepsilon} \end{Bmatrix} = T_{\dot{\varepsilon}}$$

$$T_{\dot{\varepsilon}} = \begin{Bmatrix} \dot{\varepsilon}_1 & 0 & 0 \\ 0 & \dot{\varepsilon}_2 & 0 \\ 0 & 0 & \dot{\varepsilon}_3 \end{Bmatrix}$$

Invarijante tenzora brzine deformacije

$$J_1(T_{\dot\varepsilon}) = J_1(D_{\dot\varepsilon}) \equiv \dot\varepsilon_x + \dot\varepsilon_y + \dot\varepsilon_z = \dot\varepsilon_1 + \dot\varepsilon_2 + \dot\varepsilon_3 = 0$$

$$J_2(T_{\dot\varepsilon}) = J_2(D_{\dot\varepsilon}) \equiv \dot\varepsilon_x\dot\varepsilon_y + \dot\varepsilon_y\dot\varepsilon_z + \dot\varepsilon_z\dot\varepsilon_x -$$

$$-\frac{1}{4}\left(\dot\gamma_{xy}^2 + \dot\gamma_{yz}^2 + \dot\gamma_{zx}^2\right) = \dot\varepsilon_1\dot\varepsilon_2 + \dot\varepsilon_2\dot\varepsilon_3 + \dot\varepsilon_3\dot\varepsilon_1 =$$

$$= -\frac{1}{6}\left[(\dot\varepsilon_x - \dot\varepsilon_y)^2 + (\dot\varepsilon_y - \dot\varepsilon_z)^2 + (\dot\varepsilon_z - \dot\varepsilon_x)^2\right] - \frac{1}{4}\left(\dot\gamma_{xy}^2 + \dot\gamma_{yz}^2 + \dot\gamma_{zx}^2\right)$$

$$J_3(T_{\dot\varepsilon}) = J_3(D_{\dot\varepsilon}) \equiv \begin{vmatrix} \dot\varepsilon_x & \frac{1}{2}\dot\gamma_{xy} & \frac{1}{2}\dot\gamma_{xz} \\ \frac{1}{2}\dot\gamma_{yx} & \dot\varepsilon_y & \frac{1}{2}\dot\gamma_{yz} \\ \frac{1}{2}\dot\gamma_{zx} & \frac{1}{2}\dot\gamma_{zy} & \dot\varepsilon_z \end{vmatrix} =$$

$$= \dot\varepsilon_x\dot\varepsilon_y\dot\varepsilon_z + \frac{1}{4}\dot\gamma_{xy}\dot\gamma_{yz}\dot\gamma_{zx} - \frac{1}{4}\left(\dot\varepsilon_x\dot\gamma_{yz}^2 + \dot\varepsilon_y\dot\gamma_{zx}^2 + \dot\varepsilon_z\dot\gamma_{xy}^2\right) = \dot\varepsilon_1\dot\varepsilon_2\dot\varepsilon_3$$

Ekvivalentna brzina deformacije

$$\dot\varepsilon_e = \frac{\sqrt{2}}{3}\sqrt{(\dot\varepsilon_x - \dot\varepsilon_y)^2 + (\dot\varepsilon_y - \dot\varepsilon_z)^2 + (\dot\varepsilon_z - \dot\varepsilon_x)^2 + \frac{3}{2}\left(\dot\gamma_{xy}^2 + \dot\gamma_{yz}^2 + \dot\gamma_{zx}^2\right)} =$$

$$= \frac{\sqrt{2}}{3}\sqrt{(\dot\varepsilon_1 - \dot\varepsilon_2)^2 + (\dot\varepsilon_2 - \dot\varepsilon_3)^2 + (\dot\varepsilon_3 - \dot\varepsilon_1)^2} = \sqrt{\frac{2}{3}\left(\dot\varepsilon_1^2 + \dot\varepsilon_2^2 + \dot\varepsilon_3^2\right)}$$

Monotoni procesi deformisanja i njihovi uslovi

- Monotoni process - pri deformisanjua (udaljavanju ili približavanju dve tačke materijalnog tela) deformaciona šema se ne menja.
- Pri monotonom precesu deformisanja pravci glavnih osa napona i deformacija se poklapaju. Kod nemonotnih procesa to nije slučaj.

➢ Uslovi:

- Materijalno vlakna koje se u posmatranoj etapi najbrže izdužuje (skraćuje) i u svim prethodnim etapama se najbrže izduživalo (skraćivalo), odnosno da ose glavnih brzina deformacije tokom čitavog procesa poklapaju sa istim vlaknima.
- Da se koeficijenat brzine deformacije ne menja tokom procesa.

$$\nu_{\dot{\varepsilon}} = \frac{\dot{\varepsilon}_2 - \dfrac{\dot{\varepsilon}_1 + \dot{\varepsilon}_3}{2}}{\dfrac{\dot{\varepsilon}_1 - \dot{\varepsilon}_3}{2}} = \frac{2\dot{\varepsilon}_2 - \dot{\varepsilon}_1 - \dot{\varepsilon}_3}{\dot{\varepsilon}_1 - \dot{\varepsilon}_3}$$

Veza napon-deformacija (područje elastičnosti)

Generalisani Hooke-ov zakon $\sigma = c_2 \cdot \varepsilon = E \cdot \varepsilon$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \mu_p\left(\sigma_y + \sigma_z\right)\right]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \mu_p\left(\sigma_z + \sigma_x\right)\right]$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \mu_p\left(\sigma_x + \sigma_y\right)\right]$$

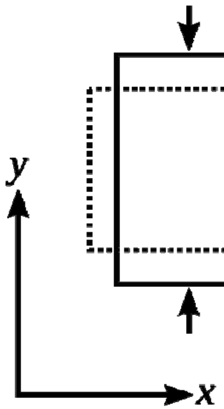
$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{2\left(1+\mu_p\right)}{E}\tau_{xy}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G} = \frac{2\left(1+\mu_p\right)}{E}\tau_{yz}$$

$$\gamma_{zx} = \frac{\tau_{zx}}{G} = \frac{2\left(1+\mu_p\right)}{E}\tau_{zx}$$

E – modul elastičnosti
G – modul smicanja

$\mu_p = -\dfrac{\varepsilon_d}{\varepsilon_l}$ *– Poisson-ov koeficijent*



$$\frac{\varepsilon_x - \varepsilon_y}{\sigma_x - \sigma_y} = \frac{\varepsilon_y - \varepsilon_z}{\sigma_y - \sigma_z} = \frac{\varepsilon_z - \varepsilon_x}{\sigma_z - \sigma_x} = \frac{\gamma_{xy}}{2\tau_{xy}} = \frac{\gamma_{yz}}{2\tau_{yz}} = \frac{\gamma_{zx}}{2\tau_{zx}} = \frac{3}{2}\frac{\varepsilon_e}{\sigma_e}$$

Robert Hooke

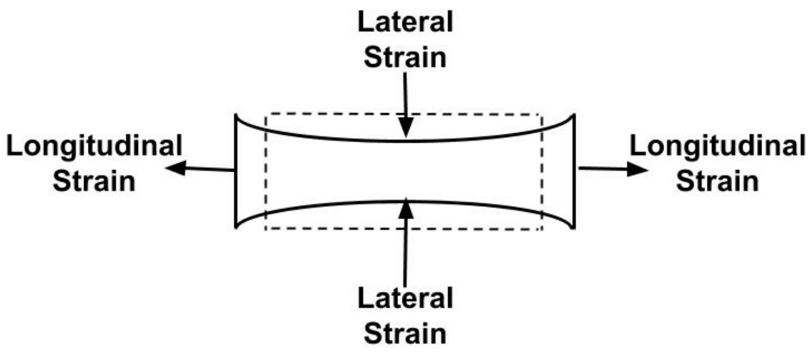
c. 1680 portrait conjectured to be Hooke[1]

Born	28 July [O.S. 18 July] 1635 Freshwater, Isle of Wight, England
Died	March 3, 1703 (aged 67) London, England
Nationality	English
Alma mater	Wadham College, Oxford
Known for	Hooke's law Microscopy Coining the term 'cell'
Scientific career	
Fields	Physics and chemistry
Institutions	Oxford University
Academic advisors	Robert Boyle
Influences	Richard Busby

Signature

Veza napon-deformacija (područje elastičnosti)

$$\mu_p = -\frac{\varepsilon_l}{\varepsilon_d}$$ *– Poisson-ov koeficijent*



$$\text{Poisson's Ratio} = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}}$$

– na sobnoj temperaturi $0 < \mu_p \leq 0{,}5$
– za čelik $\mu_p = 1/3$ (sobna temp)
$\mu_p = 0{,}5$ (temp topljenja)
– za SL $\mu_p = 0{,}11 \div 0{,}2$

– $\mu_p = 0{,}5$ – na granici tečenja

Siméon Poisson

Siméon Denis Poisson (1781–1840)

Born	21 June 1781 Pithiviers, Orléanais, Kingdom of France (present-day Loiret, France)
Died	25 April 1840 (aged 58) Sceaux, Hauts-de-Seine, July Monarchy
Nationality	French
Alma mater	École Polytechnique
Known for	Poisson process Poisson equation Poisson kernel Poisson distribution Poisson bracket Poisson algebra Poisson regression Poisson summation formula Poisson's spot Poisson's ratio Poisson zeros Conway–Maxwell–Poisson distribu Euler–Poisson–Darboux equation

Veza napon-deformacija (područje plastičnosti)

U opštem slučaju, u području plastičnosti nije moguće uspostaviti univerzalnu (jedinstvenu) vezu između komponenti napona deformacija zbog nepovratnosti procesa plastičnog deformisanja i činjenice da jednom deformacionom stanju može odgovarati više raličitih naponskih stanja!!!

- Teorija malih elasto-plastičnih deformacija – povezuje napone i male deformacije. Devijatori deformacija i napona su koaksijalni (proporcionalni) i slični.

$$\boxed{D_\varepsilon = \psi \cdot D_\sigma} \qquad \psi = \frac{3}{2}\frac{\varepsilon_e}{\sigma_e}$$ – *skalarni koeficijent (uvek pozitivan)*

$$\begin{Bmatrix} \varepsilon_x - \varepsilon_m & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{yx} & \varepsilon_y - \varepsilon_m & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{zx} & \frac{1}{2}\gamma_{zy} & \varepsilon_z - \varepsilon_m \end{Bmatrix} = \Psi \begin{Bmatrix} \sigma_x - \sigma_m & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y - \sigma_m & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z - \sigma_m \end{Bmatrix}$$

Gornja jednakost važi kako za slučaj elastičnih tako i malih elato-plastičnih deformacija (elastične i plastične deformacije su istog reda)!!
Koeficijent ψ je u oblasti elastičnosti konstantan, a u oblasti elasto-plastičnih deformacija promenjiv, tj. menja se od tačke do tačke tela, a zavisi i od stepena ostvarene deformacije!!!!!

Veza napon-deformacija (područje plastičnosti)

U slučaju velih plastičnih deformacija, zbog nehomogenosti procesa deformisanja menjaju se smerovi devijatora, tj. devijatori deformacija i napona nisu koaksijalni!!!

- Teorija velikih plastičnih deformacija (teorija priraštaja plastičnih deformacija)
  Devijator malih priraštaja deformacija ($D_{d\varepsilon}$) i devijator napona su koaksijalni i slični.

$$\boxed{D_{d\varepsilon} = d\lambda \cdot D_{\sigma}} \qquad d\lambda = \frac{3}{2}\frac{d\varepsilon_e}{\sigma_e}$$ – *skalarni koeficijent (uvek pozitivan)*

Koeficijent *dλ* se određuje isključivo eksperimentalnim putem, a zavisti od stepena deformacije, temperature, brzine deformacije, koordinata posmatrane tačke i vrste materijala.

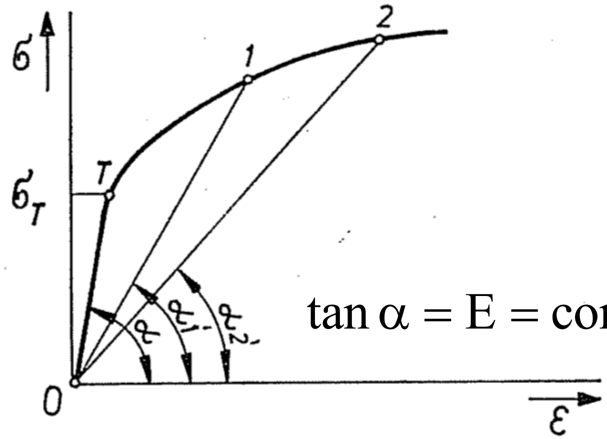
$$d\varepsilon_x = d\lambda\left(\sigma_x - \sigma_m\right) \qquad d\gamma_{xy} = 2d\lambda\tau_{xy}$$

$$d\varepsilon_y = d\lambda\left(\sigma_y - \sigma_m\right) \qquad d\gamma_{yz} = 2d\lambda\tau_{yz}$$

$$d\varepsilon_z = d\lambda\left(\sigma_z - \sigma_m\right) \qquad d\gamma_{zx} = 2d\lambda\tau_{zx}$$

$$\frac{d\varepsilon_x}{\sigma_x - \sigma_m} = \frac{d\varepsilon_y}{\sigma_y - \sigma_m} = \frac{d\varepsilon_z}{\sigma_z - \sigma_m} = \frac{2\gamma_{xy}}{\tau_{xy}} = \frac{2\gamma_{yz}}{\tau_{yz}} = \frac{2\gamma_{zx}}{\tau_{zx}} = d\lambda$$

Veza napon-deformacija
(područje plastičnosti)



$$\tan\alpha = E = \mathrm{const.}$$

$$\sigma = \sigma_e = \tan\alpha' \cdot \varepsilon = E' \cdot \varepsilon$$

E'≠0 – modul plastičnosti

$$\frac{d\varepsilon_x}{d_x} = \frac{d\varepsilon_y}{d_y} = \frac{d\varepsilon_z}{d_z} = \frac{d\gamma_{xy}}{2\tau_{xy}} = \frac{d\gamma_{yz}}{2\tau_{yz}} = \frac{d\gamma_{zx}}{2\tau_{zx}} = d\lambda$$

$$\nu_\sigma = \nu_{\dot\varepsilon}$$

$$\frac{\dot\varepsilon_x}{d_x} = \frac{\dot\varepsilon_y}{d_y} = \frac{\dot\varepsilon_z}{d_z} = \frac{\dot\gamma_{xy}}{2\tau_{xy}} = \frac{\dot\gamma_{yz}}{2\tau_{yz}} = \frac{\dot\gamma_{zx}}{2\tau_{zx}} = \lambda'$$

$$\frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3} = \frac{2\dot\varepsilon_2 - \dot\varepsilon_1 - \dot\varepsilon_3}{\dot\varepsilon_1 - \dot\varepsilon_3}$$

$$\frac{\dot\varepsilon_1}{\sigma_1 - \sigma_m} = \frac{\dot\varepsilon_2}{\sigma_2 - \sigma_m} = \frac{\dot\varepsilon_3}{\sigma_3 - \sigma_m} = \lambda' = \frac{3}{2E'}$$

$$\frac{\sigma_2 - \sigma_m}{\sigma_1 - \sigma_3} = \frac{\dot\varepsilon_2}{\dot\varepsilon_1 - \dot\varepsilon_3}$$

Prosto ili proporcionalno naprezanje

- Spoljašne sile rastu proporcionalno jednom opštem parametru – komponente tenzora napona proporcionalno se menjaju u skaldu sa tim opštim parametrom.
- Ne menja se koeficijent napona.
- Poklapaju se ose glavnih napona i deformacija.
- Podela složenijih procesa na faze u kojima postoji proporcionalno deformisanje
- Proporcionalno deformisanje vodi ka monotonom deformisanju - ekvivalentni procesi!!

Velike (konačne) deformacije

$$\frac{\varphi_x-\varphi_y}{\sigma_x-\sigma_y}=\frac{\varphi_y-\varphi_z}{\sigma_y-\sigma_z}=\frac{\varphi_z-\varphi_x}{\sigma_z-\sigma_x}=\frac{\varphi_1-\varphi_2}{\sigma_1-\sigma_2}=\frac{\varphi_2-\varphi_3}{\sigma_2-\sigma_3}=\frac{\varphi_3-\varphi_1}{\sigma_3-\sigma_1}=\lambda$$

Male deformacije

$$\frac{\varepsilon_x-\varepsilon_y}{\sigma_x-\sigma_y}=\frac{\varepsilon_y-\varepsilon_z}{\sigma_y-\sigma_z}=\frac{\varepsilon_z-\varepsilon_x}{\sigma_z-\sigma_x}=\frac{\varepsilon_1-\varepsilon_2}{\sigma_1-\sigma_2}=\frac{\varepsilon_2-\varepsilon_3}{\sigma_2-\sigma_3}=\frac{\varepsilon_3-\varepsilon_1}{\sigma_3-\sigma_1}=\lambda$$

$$\lambda=\frac{3}{2}\frac{\varepsilon_e}{\sigma_e}=\frac{3}{2}\frac{\varphi_e}{K} \qquad \lambda=\frac{3}{2E'}=\frac{1}{2G'}$$